Sparse and Plausible Counterfactual Explanations

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Counterfactual Explanations (CFEs)

Explainabile Artificial Intelligence (XAI)

- Use of inherently interpretable and transparent machine learning (ML) models or generating post-hoc explanations for opaque models
- Ensure decisions produced by the ML system are not biased against a particular demographic group of individuals

Counterfactual Explanations (CFEs)

- Specific class of XAI in ML
- Provide a link between what could have happened had input to a model been changed in a particular way
  - Do not answer the *why* the model made a prediction - XAI
  - Provide suggestions to achieve the desired outcome
- Appealing in high-impact areas such as finance and healthcare
  - Credit lending
  - Talent sourcing
  - Parole
  - Medical treatment

Setup

Classification setting

- $\mathcal{X}^n$ – input space of features
- $\mathcal{Y}$ – output space of labels
- Learned function $f : \mathcal{X}^n \to \mathcal{Y}$

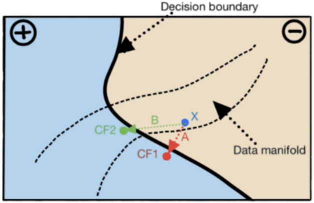


Figure 1: Two possible CFE paths for a datapoint $\mathbf{x}$ (shortest path (red) vs. path adhering closest to the manifold (green) of training data).

Credit lending example

- Alice seeks a home mortgage loan
- ML classifier considers Alice's feature vector $\{Income, CreditScore, Education, Age\}$
- Alice is denied the loan
  - Why the loan was denied? - XAI
    - *CreditScore* was too low
  - What can she do differently so that the loan will be approved in the future? - **CFE**
    - Increase *Income* by $10K
    - Get a master's degree
    - A combination of both

CFE Definition

1. CFEs should quantify a relatively *small change* in only a *few features*
   - E.g., Increase *only* Alice's income (e.g. by \$10K instead of \$50K)
2. CFEs should be *realistic* and *actionable*
   - E.g., Alice cannot decrease her age by ten years

Definition ([Dan+20])

Let $f : \mathcal{X}^n \to \mathcal{Y}$ be a prediction function. A CFE $\mathbf{x}^{'}$ for an observation $\mathbf{x}^*$ is defined as a data point fulfilling the following:

1. *(Validity)* its prediction $f(\mathbf{x}^{'})$ is close to the desired $\mathcal{Y}$,
2. *(Proximity)* it is close to $\mathbf{x}^*$ in $\mathcal{X}$,
3. *(Sparsity)* it differs from $\mathbf{x}^*$ only in a few features,
4. *(Plausibility)* it is a plausible data point according to the probability distribution $\mathbf{P}_{\mathcal{X}}$.

- For classification models
  - f returns the probability for a user-selected class
  - $\mathcal{Y}$ is the desired probability (range)

1st approach: Sparse and Imperceptible Adversarial Attacks with Convex Hull Witness Penalty

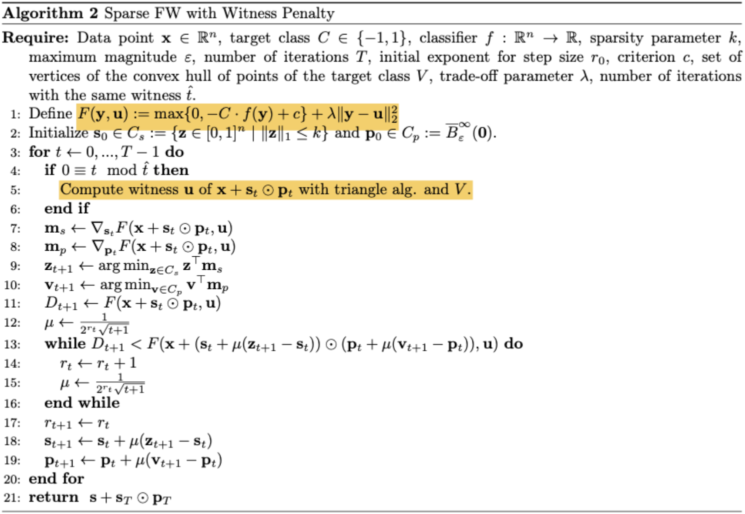
- *Validity*, *Proximity*, and *Sparsity* via Adversarial Attacks
  - Utilize the extensive literature on sparse and impreceptible adversarial attacks
    - E.g., SAIF: Sparse Adversarial and Imperceptible Attack Framework [Imt+22]
  - Set the *change* $\mathbf{w} := \mathbf{x}^{'} - \mathbf{x}^{*}$ by $\mathbf{w} = \mathbf{s} \odot \mathbf{p}$
    - $\mathbf{s}$ sparsity mask
    - $\mathbf{p}$ change magnitude
  - Optimize simultaneously for sparsity (1−norm of $\mathbf{s}$, relaxation of 0−norm) and proximity (∞-norm of $\mathbf{p}$) using Frank-Wolfe (FW) on the following problem

$$
\begin{aligned}
\underset{\mathbf{s},\mathbf{p}}{\arg\min} \quad & \max\{0, -C \cdot f(\mathbf{x}^{*} + \mathbf{s} \odot \mathbf{p}) + c\} \\
\text{s.t.} \quad & \|\mathbf{s}\|_1 \leq k, \mathbf{s} \in [0,1]^n \\
& \|\mathbf{p}\|_\infty \leq \epsilon
\end{aligned}
$$

    - $C \in \{-1, 1\}$ is the target class
    - k is a sparsity parameter
    - ϵ is maximum magnitude

- *Plausibility* by requiring the CFE to lie in the convex hull of correctly classified points
  - Computing the vertices of the convex hull using `qhull` in high-dimensions is hard (?)
  - Instead add a penalty term for the distance to the witness of convex hull produced by the triangle algorithm [AKZ18]

SAIF with Witness Penalty Algorithm



Algorithm 2 Sparse FW with Witness Penalty

```
Require: Data point $\mathbf{x} \in \mathbb{R}^n$, target class $C \in \{-1, 1\}$, classifier $f : \mathbb{R}^n \to \mathbb{R}$, sparsity parameter $k$,
    maximum magnitude $\varepsilon$, number of iterations $T$, initial exponent for step size $r_0$, criterion $c$, set of
    vertices of the convex hull of points of the target class $V$, trade-off parameter $\lambda$, number of iterations
    with the same witness $\hat{t}$.
Define $F(\mathbf{y}, \mathbf{u}) := \max\{0, -C \cdot f(\mathbf{y}) + c\} + \lambda \|\mathbf{y} - \mathbf{u}\|_2^2$
Initialize $\mathbf{s}_0 \in C_s := \{\mathbf{z} \in [0,1]^n \mid \|\mathbf{z}\|_1 \leq k\}$ and $\mathbf{p}_0 \in C_p := \overline{B}_\varepsilon^\infty(\mathbf{0})$.
for $t \leftarrow 0, ..., T-1$ do
    if $0 \equiv t \mod \hat{t}$ then
        Compute witness $\mathbf{u}$ of $\mathbf{x} + \mathbf{s}_t \odot \mathbf{p}_t$ with triangle alg. and $V$.
    end if
    $\mathbf{m}_s \leftarrow \nabla_{\mathbf{s}_t} F(\mathbf{x} + \mathbf{s}_t \odot \mathbf{p}_t, \mathbf{u})$
    $\mathbf{m}_p \leftarrow \nabla_{\mathbf{p}_t} F(\mathbf{x} + \mathbf{s}_t \odot \mathbf{p}_t, \mathbf{u})$
    $\mathbf{z}_{t+1} \leftarrow \arg\min_{\mathbf{z} \in C_s} \mathbf{z}^\top \mathbf{m}_s$
    $\mathbf{v}_{t+1} \leftarrow \arg\min_{\mathbf{v} \in C_p} \mathbf{v}^\top \mathbf{m}_p$
    $D_{t+1} \leftarrow F(\mathbf{x} + \mathbf{s}_t \odot \mathbf{p}_t, \mathbf{u})$
    $\mu \leftarrow \frac{1}{2^{r_t}\sqrt{t+1}}$
    while $D_{t+1} < F(\mathbf{x} + (\mathbf{s}_t + \mu(\mathbf{z}_{t+1} - \mathbf{s}_t)) \odot (\mathbf{p}_t + \mu(\mathbf{v}_{t+1} - \mathbf{p}_t)), \mathbf{u})$ do
        $r_t \leftarrow r_t + 1$
        $\mu \leftarrow \frac{1}{2^{r_t}\sqrt{t+1}}$
    end while
    $r_{t+1} \leftarrow r_t$
    $\mathbf{s}_{t+1} \leftarrow \mathbf{s}_t + \mu(\mathbf{z}_{t+1} - \mathbf{s}_t)$
    $\mathbf{p}_{t+1} \leftarrow \mathbf{p}_t + \mu(\mathbf{v}_{t+1} - \mathbf{p}_t)$
end for
return $\mathbf{s} + \mathbf{s}_T \odot \mathbf{p}_T$
```

Potential issues with this approach

- *Sparsity* and *Plausibility* are conflicting goals [Dan+20]
- Convex hull covers a lot of empty space of low data density in high dimensions

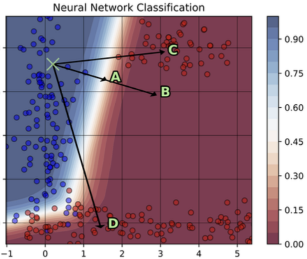


Figure 2: Four viable CFEs of ×, all satisfying the validity. A minimizes for proximity and B has a large classification margin (validity). Nevertheless, both A and B lie in a low density region. C and D lie in high-density regions and have a large classification margin, but are less sparse. However, connection between × and D is via a high-density path, hence it is feasible for the original instance to be transformed into D despite C being simply closer.

- Does our 1^{st} approach result in CFEs in low density regions?
  - The witness penalty usually results in points closer to the vertices of the convex hull

2^{nd} approach: Accelerated Proximal Gradient (APG) Method

- *Plausibility* via training a KDE term for the target class
- *Sparsity* via $0-$ norm
- *Proximity* via Gower distance

$$\arg\min_{\mathbf{w}} \quad \max\{0, -C \cdot f(\mathbf{x}^* + \mathbf{w}) + c\} + \lambda \|\mathbf{w}\|_0 + \mathrm{Gow}(\mathbf{w}) - KDE(\mathbf{x}^* + \mathbf{w}, t)$$

  - t is the target class
  - Gower distance is defined by

$$\mathrm{Gow}(\mathbf{w}) := \frac{1}{n} \sum_{i=1}^{n} \delta_{\mathrm{Gow}}(w_i) \in [0,1], \quad \delta_{\mathrm{Gow}}(w_i) := \begin{cases} \frac{1}{\mathcal{A}_i} |w_i|, & \text{if } \mathbf{x}_j \text{ is numerical} \\ \mathbb{I}_{\mathbf{x}_j \neq \mathbf{x}'_j}, & \text{if } \mathbf{x}_j \text{ is categorical} \end{cases}$$

    - *Actionability* - $\mathcal{A}_i$ the value range for feature i, extracted from the observed dataset (or given by the user)
    - For numerical data, we have box constraints ($|w_i| \leq \mathcal{A}_i$)
    - Use the indicator function such that

$$I_{[-\mathcal{A}_i, \mathcal{A}_i]}(w_i) := \begin{cases} 0, & \text{if } w_i \in [-\mathcal{A}_i, \mathcal{A}_i] \\ +\infty, & \text{otherwise} \end{cases}$$

- New problem for numerical data

$$\arg\min_{\mathbf{w}} \quad \max\{0, -C \cdot f(\mathbf{x}^* + \mathbf{w}) + c\} + \lambda \|\mathbf{w}\|_0 + I_{[-\mathcal{A}, \mathcal{A}]}(\mathbf{w}) - KDE(\mathbf{x}^* + \mathbf{w}, t)$$

2^{nd} approach: Accelerated Proximal Gradient (APG) Method

- Denote $h(\mathbf{x}^* + \mathbf{w}) := \max\{0, -C \cdot f(\mathbf{x}^* + \mathbf{w}) + c\} - KDE(\mathbf{x}^* + \mathbf{w}, t)$
- Do a quadratic approximation $\tilde{h}_L(\mathbf{x}^* + \mathbf{w})$ to $h(\mathbf{x}^* + \mathbf{w})$
- Replace $\nabla^2 h(\mathbf{x}^* + \mathbf{w})$ by $\frac{L}{2} I$

$$\begin{aligned}
\mathbf{w}^{k+1} &= \arg\min_{\mathbf{w}} \tilde{h}_L(\mathbf{x}^* + \mathbf{w}) + \lambda \|\mathbf{w}\|_0 + I_{[-\mathcal{A},\mathcal{A}]}(\mathbf{w}) \\
&= \arg\min_{\mathbf{w}} \nabla_{\mathbf{w}} h(\mathbf{x}^* + \mathbf{w}^k)^T (\mathbf{w} - \mathbf{w}^k) + \frac{L}{2} \|\mathbf{w} - \mathbf{w}^k\|_2^2 + \lambda \|\mathbf{w}\|_0 + I_{[-\mathcal{A},\mathcal{A}]}(\mathbf{w}) \\
&= \arg\min_{\mathbf{w}} \frac{L}{2} \|[\mathbf{w}^k - \frac{1}{L} \nabla_{\mathbf{w}} h(\mathbf{x}^* + \mathbf{w}^k)] - \mathbf{w}\|_2^2 + \lambda \|\mathbf{w}\|_0 + I_{[-\mathcal{A},\mathcal{A}]}(\mathbf{w}) \qquad (1)
\end{aligned}$$

- How do we compute $\nabla_{\mathbf{w}} h(\mathbf{x}^* + \mathbf{w}^k)$?
  - In case of the Gaussian normal kernel [Rac+08]

    $$KDE(\mathbf{x}^* + \mathbf{w}, t) := \frac{1}{n} \sum_{i=1}^{n} e^{-\|\mathbf{w} - \mathbf{b}_i\|_2^2 / 2\sigma^2}$$

    where $\mathbf{b}^i := -(\mathbf{x}^* - \mathbf{x}^i)$ for correctly classified points $\mathbf{x}^i$
  - Then

    $$\nabla_{\mathbf{w}} KDE(\mathbf{x}^* + \mathbf{w}, t) = -\frac{1}{n\sigma^2} \sum_{i=1}^{n} (\mathbf{w} - \mathbf{b}^i) e^{-\|\mathbf{w} - \mathbf{b}_i\|_2^2 / 2\sigma^2}$$

  - Instead of backpropagating the whole h function, use the closed-form solution for the KDE term

2^{nd} approach: Accelerated Proximal Gradient (APG) Method

- Let $g(\mathbf{w}) := \lambda\|\mathbf{w}\|_0 + I_{[-\mathcal{A},\mathcal{A}]}(\mathbf{w})$
- Solution to Eqn. (1) is denoted as

$$\operatorname{Prox}_{\frac{1}{L}}(\mathbf{w}^k - \frac{1}{L}\nabla_{\mathbf{w}} h(\mathbf{x}^* + \mathbf{w}^k)) = \arg\min_{\mathbf{w}} \frac{L}{2}\|[\mathbf{w}^k - \frac{1}{L}\nabla_{\mathbf{w}} h(\mathbf{x}^* + \mathbf{w}^k)] - \mathbf{w}\|_2^2 + \lambda\|\mathbf{w}\|_0 + I_{[-\mathcal{A},\mathcal{A}]}(\mathbf{w}) \qquad (2)$$

- Obtain the solution explicitly [ZCW21]
  - Let

$$S_L(\mathbf{w}) = \mathbf{w} - \frac{1}{L}\nabla_{\mathbf{w}} h(\mathbf{x}^* + \mathbf{w}), \quad \forall \mathbf{w} \in [-\mathcal{A}, \mathcal{A}]$$

$$\Pi_{[-\mathcal{A},\mathcal{A}]}(\mathbf{w}) = \arg\min_{\mathbf{w}}\{\|\mathbf{y} - \mathbf{w}\| : \mathbf{y} \in [-\mathcal{A}, \mathcal{A}]\}, \quad \forall \mathbf{w} \in \mathbb{R}^n$$

- Solution to Eqn.(2) for $i = 1, 2, ..., n$ is given by [XZ13]

$$w_i^{k+1} = \begin{cases} [\Pi_{[-\mathcal{A},\mathcal{A}]}(S_L(w^k))]_i, & \text{if } [S_L(w^k)]_i^2 - [\Pi_{[-\mathcal{A},\mathcal{A}]}(S_L(w^k)) - S_L(w^k)]_i^2 > \frac{2\lambda}{L} \\ 0, & \text{otherwise} \end{cases}$$

Can we drop the *Validity* requirement?

Classification setting

- Generating process $\psi = (\mathcal{X}^n, \mathcal{Y}, p)$
  - $p : \mathcal{X}^n \times \mathcal{Y} \mapsto \mathbb{R}_+$ denotes joint density
  - $\{\mathbf{x} \in \mathcal{X}^n | \quad p(\mathbf{x}, y) \geq \delta\}$ closed for all $\delta > 0, y \in \mathcal{Y}$

Theorem (Model free $\delta-$plausible CFEs under zero risk classifiers [AH20])

Let $\mathcal{F}$ be the set of all classifiers $f : \mathcal{X}^n \to \mathcal{Y}$ that have zero risk on the generating process ψ, i.e., $f \in \mathcal{F} \Leftrightarrow \mathbb{E}_{\mathbf{x},y\sim p}[\mathbb{1}(f(\mathbf{x} \neq y))] = 0$. Then the following holds $\forall f \in \mathcal{F}, (\mathbf{x}, y^{cfe}) \in \mathcal{X}^n \times \mathcal{Y} \backslash \{y\}$:

$$\begin{aligned} &\arg\min_{\mathbf{w}} \ \theta(\mathbf{w}) \ \text{ s.t. } \ f(\mathbf{x}^{'}) = y^{cfe} \wedge p(\mathbf{x}^{'}, y^{cfe}) \geq \delta \\ \Leftrightarrow \ &\arg\min_{\mathbf{w}} \ \theta(\mathbf{w}) \ \text{ s.t. } \ p(\mathbf{x}^{'}, y^{cfe}) \geq \delta \end{aligned}$$

- $\theta : \mathcal{X}^n \times \mathcal{X}^n \mapsto \mathbb{R}_+$ a distance metric in $\mathcal{X}^n$

3rd approach: $k-$Nearest Neighbors ($k-$NN) Approach

- Instead of training a KDE, simply consider $k-$Nearest Neighbors ($k-$NN) of $\mathbf{x}^*$
- Denote $f(\mathbf{x}^* + \mathbf{w}) := \max\{0, -C \cdot f(\mathbf{x}^* + \mathbf{w}) + c\}$ and rewrite

$$\underset{\mathbf{w}}{\arg\min} \quad f(\mathbf{x}^* + \mathbf{w}) + \lambda\|\mathbf{w}\|_0 + I_{[-\mathcal{A},\mathcal{A}]}(\mathbf{w}) + kNN(\mathbf{x}^* + \mathbf{w}, X^{\text{obs}}) \tag{3}$$

  - $\mathbf{x}^1, ..., \mathbf{x}^k \in X^{\text{obs}}$ - k nearest observed datapoints of the target class

$$kNN(\mathbf{x}^* + \mathbf{w}, X^{\text{obs}}) := \left\|(\mathbf{x}^* + \mathbf{w}) - \sum_{i=1}^{k} \hat{\mathbf{a}}_i \mathbf{x}^i\right\|_2^2, \quad \sum_{i=1}^{k} \hat{\mathbf{a}}_i = 1, \quad \hat{\mathbf{a}}_i \geq 0.$$

  - $\hat{\mathbf{a}}_i$ calculated based on an approximation of LOF as in [Zha+23]
- Reformulate Eqn. (3) in a way that lends itself to the application of ADMM

$$\begin{aligned} \underset{\mathbf{z},\mathbf{w},\mathbf{y}}{\arg\min} \quad & f(\mathbf{x}^* + \mathbf{z}) + \lambda\|\mathbf{y}\|_0 + I_{[-\mathcal{A},\mathcal{A}]}(\mathbf{y}) + kNN(\mathbf{x}^* + \mathbf{w}, X^{\text{obs}}) \\ \text{s.t.} \quad & \mathbf{z} = \mathbf{y}, \mathbf{z} = \mathbf{w} \end{aligned} \tag{4}$$

  - $\mathbf{z}, \mathbf{y}$ are newly introduced variables

3^{rd} approach: $k-$Nearest Neighbors ($k-$NN) Approach

- Perform ADMM by minimizing the augmented Lagrangian of Eqn. (4)

$$\begin{aligned} L(\mathbf{z}, \mathbf{y}, \mathbf{w}, \mathbf{m}, \mathbf{n}) = f(\mathbf{x}^* + \mathbf{z}) + \lambda \|\mathbf{y}\|_0 + I_{[-\mathcal{A}, \mathcal{A}]}(\mathbf{y}) + kNN(\mathbf{x}^* + \mathbf{w}, X^{\text{obs}}) \\ + \mathbf{m}^\top(\mathbf{y} - \mathbf{z}) + \mathbf{n}^\top(\mathbf{w} - \mathbf{z}) + \frac{\rho}{2}\|\mathbf{y} - \mathbf{z}\|_2^2 + \frac{\rho}{2}\|\mathbf{w} - \mathbf{z}\|_2^2 \end{aligned} \tag{5}$$

  - $\mathbf{m}, \mathbf{n}$ are Lagrangian multipliers
  - ρ is a penalty parameter

$$\{\mathbf{w}^{(k+1)}, \mathbf{y}^{(k+1)}\} = \arg\min_{\mathbf{w}, \mathbf{y}} L(\mathbf{z}^{(\mathbf{k})}, \mathbf{y}, \mathbf{w}, \mathbf{m}^{(\mathbf{k})}, \mathbf{n}^{(\mathbf{k})}) \tag{6}$$

$$\mathbf{z}^{(k+1)} = \arg\min_{\mathbf{z}} L(\mathbf{z}, \mathbf{y}^{(\mathbf{k+1})}, \mathbf{w}^{(\mathbf{k+1})}, \mathbf{m}^{(\mathbf{k})}, \mathbf{n}^{(\mathbf{k})}) \tag{7}$$

$$\begin{aligned} \mathbf{m}^{(k+1)} &= \mathbf{m}^{(k)} + \rho(\mathbf{u}^{(k+1)} - \mathbf{z}^{(k+1)}) \\ \mathbf{n}^{(k+1)} &= \mathbf{n}^{(k)} + \rho(\mathbf{y}^{(k+1)} - \mathbf{z}^{(k+1)}) \end{aligned} \tag{8}$$

- Can we find the solution to Eqns. (6)-(8) in parallel and exactly?

w−solution

- For the **w** we have

$$\begin{aligned}\mathbf{w}^{(k+1)} &= \arg\min_{\mathbf{w}} \quad kNN(\mathbf{x}^* + \mathbf{w}, X^{\text{obs}}) + {\mathbf{n}^{(k)}}^\top(\mathbf{w} - \mathbf{z}^{(k)}) + \frac{\rho}{2}\|\mathbf{w} - \mathbf{z}^{(k)}\|_2^2 \\ &= \arg\min_{\mathbf{w}} \quad \left\|(\mathbf{x}^* + \mathbf{w}) - \sum_{i=1}^{k} \hat{\mathbf{a}}_i \mathbf{x}^i\right\|_2^2 + \frac{\rho}{2}\left\|\mathbf{w} - \mathbf{c}^{(k)}\right\|_2^2 \qquad (9)\end{aligned}$$

  - $\mathbf{c}^{(k)} = \left(\mathbf{z}^{(k)} - \frac{\mathbf{n}^{(k)}}{\rho}\right)$

- Denote $\mathbf{b}^i := -(\mathbf{x}^* - \mathbf{x}^i)$, then Eqn. (9) in 1D is equivalent to

$$\arg\min_{w} \left(x^* + w - \sum_{i=1}^{k} \hat{\mathbf{a}}_i x^i\right)^2 + \frac{\rho}{2}(w - c)^2$$

- Simply solve the resulting quadratic equation

y−solution and z−solution

- For the $\mathbf{y}$ we have

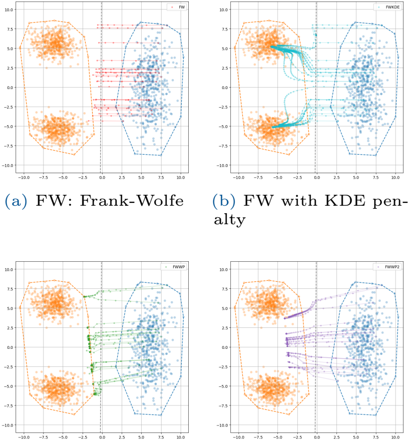
$$\begin{aligned}\mathbf{y}^{(k+1)} &= \arg\min_{\mathbf{w}} \quad \lambda\|\mathbf{y}\|_0 + I_{[-\mathcal{A},\mathcal{A}]}(\mathbf{y}) + \mathbf{m}^{(k)}{}^{\top}(\mathbf{y}-\mathbf{z}^{(k)}) + \frac{\rho}{2}\|\mathbf{y}-\mathbf{z}^{(k)}\|_2^2 \\ &= \arg\min_{\mathbf{w}} \quad \lambda\|\mathbf{y}\|_0 + I_{[-\mathcal{A},\mathcal{A}]}(\mathbf{y}) + \frac{\rho}{2}\left\|\mathbf{y}-\mathbf{d}^k\right\|_2^2 \qquad (10)\end{aligned}$$

  - $\mathbf{d}^{(k)} = \left(\mathbf{z}^{(k)} - \frac{\mathbf{m}^{(k)}}{\rho}\right)$
  - Similarly to APG, solution to Eqn. (10) for $i = 1, \ldots, n$ is given by [ZCW21]

$$w_i^{(k+1)} = \begin{cases} [\Pi_{[-\mathcal{A},\mathcal{A}]}(d_i^{(k)})]_i, & \text{if } [d_i^{(k)}]_i^2 - [\Pi_{[-\mathcal{A},\mathcal{A}]}(d_i^{(k)}) - d_i^{(k)}]_i^2 > \frac{2\lambda}{L} \\ 0, & \text{otherwise} \end{cases}$$

- For the $\mathbf{z}$ - Eqn. (7)
  - Split the function f and do a first-order Taylor expansion at the point $\mathbf{z}^k$ which yields a quadratic program which has a closed-form solution [Xu+19]

Results



(a) FW: Frank-Wolfe

(b) FW with KDE penalty

(c) FW with witness penalty

(d) Witnesses moved half way towards mean of the vertices.

Figure 3: Visualization of the iterates of non-sparse variants of our algorithms.

Further Evaluation

- *Sparsity* - the number of feature changes

$$\frac{1}{n}\sum_{j=1}^{n} \mathbb{1}\{\mathbf{x}^{'} \neq \mathbf{x}^{*}\}$$

- *Proximity* - report the average ℓ_1-norm [Zha+23] (ℓ_2- norm [TSP24]) of the CFE to the observed factual point
- *Plausibility*
  1. Compare Local Outlier Factor (LOF) metric [Bre+00] of our $k-$NN approach to [Zha+23; TSP24]
     - LOF analyzes to what extent a data point is an outlier in the data manifold
     - $LOF(\mathbf{x})$ close to 1 means $\mathbf{x}$ is an inlier
     - Larger values (especially $LOF(\mathbf{x}) > 1.5$) means $\mathbf{x}$ is an outlier
  2. Compare the log-density of CFEs under the kernel density estimator of our APG to [AH20]
- *Validity* - the ratio of the counterfactuals that actually have the desired class label to the total number of counterfactuals generated
- Average *runtime* per method

Geometrical Comparison

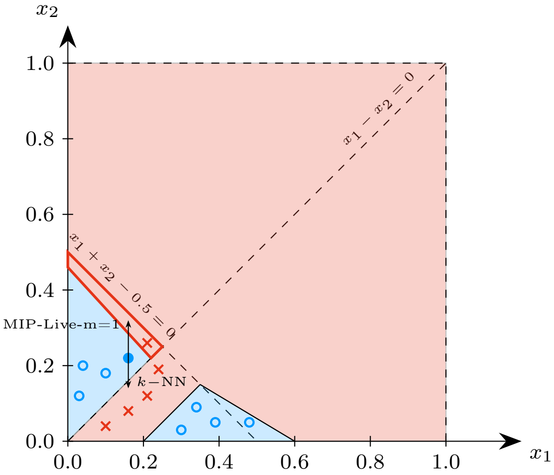


Figure 4: MIP-Live-m=1 [TSP24] vs. our $k-$NN approach. The generated CFE of our method resides in a high-density region and is sparse. MIP-Live-m=1 restricts considerably the working space - the small bounded red region, and uses only 1 neighbor for the LOF manifold adhering constraint.

Discussion and Future Work

- How do we extend our approaches to be model-agnostic?
  - Approximate the AI system with a substitute model [Gui+19]
  - Use our proposed method to generate CFEs using our substitute model
  - Study the role of substitute model used [Dan+24]
  - Simply calculate the gradients without training a substitute model
- How do we extend our approaches to include categorical variables?
  - Linearly ordered categorical data [Dhu+19]
  - One-hot encoding [Rus19]
  - GANs paper dealing with categorical data [Nem+22]
- How do we measure plausibility?
  - Log-KDE value of generated CFEs [AH20]
  - Plausibility reward function via Autoencoder reconstruction loss [BLM23]
  - The distance to $k-$nearest neighbors [Dan+20]

Discussion and Future Work

- Plausible CFEs, in general, cannot be interpreted as action recommendations
- CFEs provide hints about which alternative feature values would yield acceptance by the predictor
  - Do not guide the user on which interventions yield the desired change in the real world
  - To guide action, causal knowledge is required
- Proximity and plausibility are conflicting objectives [Dan+24]
  - Oftentimes, there is only little data close to the decision boundary, and jumping just over the boundary can lead to implausible CFEs
- *Improvement* of the underlying target is more desirable than *acceptance* by a specific predictor
  - E.g., Covid infection prediction - intervening on the symptoms may change the diagnosis (prediction), but will not affect whether someone is infected (real-world state) [KFG23]

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